Dynamic spectral density function of a dusty plasma

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(Received 28 December 1994; revised manuscript received 31 July 1995)

Dust particles in a plasma can be highly charged. The Debye screening of the dust particles can seriously affect the Coulomb collective interaction. The dynamic spectral density function of a dusty plasma in a weak turbulence and weakly coupled state has been derived by kinetic theory. A detailed analysis of the electron dynamic spectral density function in the dusty plasma shows that the Debye screening of the dust particles increases the electron density correlation of the plasma considerably. This increase is proportional to the ratio of densities n_d/n_e and the square of the dust charge number $Z_d^2=(q_0/e)^2$. A correction of electron density correlation of an order $P\Omega_c^2/\omega^2$ results because of the change of the charge on the dust particles. Here P is a parameter describing the importance of the dust charge in the plasma, Ω_c is the charging frequency of the dust particle, and ω is the fluctuation frequency of the plasma.

PACS number(s): 52.25.Dg, 52.25.Kn, 52.35.Ra, 52.35.Fp

I. INTRODUCTION

Dusty plasma occurs frequently not only in nature (interstellar clouds, solar system, the Earth's atmosphere, etc.), but also in man-made environments such as, for example, microelectronics fabrication, materials synthesis, and laboratory experiments. Recently there has been a growing interest in dusty plasma studies [1–5]. It is well known that when dust particles are immersed in a partially or fully ionized plasma, the charge characteristics of the dust particles makes it a special component of the plasma. One of the characteristics is they can be highly charged $Z_d=q_0/e\sim 10^3$ [T (eV)] [a (μm)] (see Sec. IV). Another is that the charge on the dust particles is not constant, it can fluctuate and also change if the dust density or plasma conditions change. This should significantly influence on the collective properties of the plasma, where the Coulomb interactions play a dominant role. Indeed, in some cases a dusty plasma exhibits special collective effects such as strong scattering of electromagnetic waves by dust in the Earth's mesosphere [6], the crystallization of a dusty plasma in experiments [7], the dust self-organization behavior in a rf discharge plasma [8], etc.

The dynamic spectrum density function $S(\omega,\mathbf{k})$ is a spectral function of the density fluctuation correlation. In a uniform and stationary system it can be written as

$$S_\alpha(\omega,\mathbf{k})=\frac{1}{n_\alpha}\int d\Delta\mathbf{r}\int d\Delta t\,\langle \delta n_\alpha(\mathbf{r},t)\delta n_\alpha(\mathbf{r}+\Delta\mathbf{r},t+\Delta t)\rangle \times\exp[i\mathbf{k}\cdot\Delta\mathbf{r}-i\omega\Delta t] \tag{1}$$

or

$$\langle \delta n_\alpha(\mathbf{r},t)\delta n_\alpha(\mathbf{r}+\Delta\mathbf{r},t+\Delta t)\rangle =\frac{n_\alpha}{(2\pi)^4}\int d\mathbf{k}\int d\omega\, S_\alpha(\omega,\mathbf{k})\exp[-i\mathbf{k}\cdot\Delta\mathbf{r}+i\omega\Delta t]\ . \tag{2}$$

The dynamic spectrum density function $S(\omega,\mathbf{k})$ represents the power spectrum of the density fluctuation in the frequency and wave vector space, which can give the information of the collective interaction of the many-body system such as a plasma [9]. In present paper the dynamic spectrum density function of a dusty plasma has been studied in a kinetic description with the following assumptions.

(i) A static dusty plasma system is assumed. This means that there is an external source in the system to balance the plasma particles lost in charging the dust. Such a case could be, for example, a dust cloud embedded in an infinite plasma.

(ii) The dusty plasma is assumed in a state of weak turbulence and weak coupling. We therefore linearize the dynamic equations in which the pair correlation collision term will be neglected.

(iii) The dust particles in the system are spherical with the same radius.

II. CHARGE FLUCTUATION OF DUST IN A PLASMA

Unlike the plasma particles the dust particles are not fully described by their location in $\{\mathbf{r},\mathbf{v}\}$ space because their charge must also be specified. The phase space of the dust particles must therefore be increased to include the dust charge q. The distribution function of dust particles N_d in a phase space $\{\mathbf{r},\mathbf{v},q\}$ satisfies a dynamic equation:

$$\frac{\partial}{\partial t}N_d+\mathbf{v}_d\cdot\frac{\partial}{\partial \mathbf{r}}N_d+\frac{q}{M_d}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}_d}N_d +\frac{\partial}{\partial q}[I(q)N_d]=0\ . \tag{3}$$

The currents to a dust particle are

$$I(q)=\sum_\alpha\int\sigma_\alpha(q,v_\alpha)e_\alpha v_\alpha N_\alpha(\mathbf{v}_\alpha)d\mathbf{v}_\alpha\ , \tag{4}$$

where $\alpha=\{e,i\}$, and the collision cross section is

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$$\sigma_\alpha(q,v_\alpha)=\begin{cases}\pi\alpha^2\left[1-\frac{2qe_\alpha}{am_\alpha v_\alpha^2}\right] & \text{for } \frac{2qe_\alpha}{am_\alpha v_\alpha^2}\le 1\\ 0 \quad \text{for } \frac{2qe_\alpha}{am_\alpha v_\alpha^2}\ge 1\ , \end{cases} \tag{5}$$

e_α, m_α is the charge and mass of the particles, and a is the radius of the dust particle [10]. The ensemble average gives

$$\langle N_d\rangle = F_d$$

and

$$\langle N_\alpha\rangle = F_\alpha\ .$$

Let $N_d=F_d+\delta f_d$, $N_\alpha=F_\alpha+\delta f_\alpha$, where δf_d and δf_α refer to the fluctuation part of the distribution of dust component and of the plasma, respectively. An equation of the fluctuation of dust particles is found by the linearizing of Eq. (3) with the condition $\delta f_d \ll F_d$ and $\delta f_\alpha \ll F_\alpha$. This gives

$$\frac{\partial}{\partial t}\delta f_d+\mathbf{v}_d\cdot\frac{\partial}{\partial \mathbf{r}}\delta f_d+\frac{q}{M_d}\delta\mathbf{E}\cdot\frac{\partial F_d}{\partial \mathbf{v}_d} + \frac{\partial}{\partial q}\sum_\alpha e_\alpha\int\sigma_\alpha(q,v_\alpha)v_\alpha(\delta f_\alpha F_d+\delta f_d F_\alpha)d\mathbf{v}_\alpha=0\ . \tag{6}$$

In an equilibrium state, the charge on a dust particle is found from the equation

$$I_0(q_0)\equiv\sum_\alpha\int\sigma_\alpha(q_0,v_\alpha)e_\alpha v_\alpha F_\alpha(\mathbf{v}_\alpha)d\mathbf{v}_\alpha=0\ . \tag{7}$$

It can be seen that the equilibrium charge q_0 is determined by the density of plasma n_α, its temperature T_α, and the radius α of the dusty particle. On the assumption that all dust particles in the plasma are of one size, the equilibrium distribution function of the dust particles can be written as

$$F_d(q,\mathbf{v}_d)=\delta(q-q_0)\Phi_d(\mathbf{v}_d)\ . \tag{8}$$

Now we define a function

$$\delta f_d^\rho(\mathbf{r},\mathbf{v}_d,t)\equiv\int(q-q_0)\delta f_d(\mathbf{r},\mathbf{v}_d,q,t)dq\ . \tag{9}$$

A charge density fluctuation due to the changing charge on the dust particles can be found from

$$\Delta\rho_d(\mathbf{r},t)=\int\delta f_d^\rho(\mathbf{r},\mathbf{v}_d,t)d\mathbf{v}_d\ . \tag{10}$$

Multiplying Eq. (6) by $(q-q_0)$, in which F_d is substituted by Eq. (8), and integrating over the q space, we get

$$\frac{\partial}{\partial t}\delta f_d^\rho+\mathbf{v}_d\cdot\frac{\partial}{\partial\mathbf{r}}\delta f_d^\rho+\Omega_c\delta f_d^\rho + \Phi_d(\mathbf{v}_d)\sum_\alpha e_\alpha\int\sigma_\alpha(q_0,v_\alpha)v_\alpha\delta f_\alpha d\mathbf{v}_\alpha=0\ . \tag{11}$$

In the above derivation Eq. (4) has been used in a form, which is expanded at $q=q_0$ as

$$I(q)=-\Omega_c(q-q_0)\ ,$$

where

$$\Omega_c=-\left.\frac{\partial I(q)}{\partial q}\right|_{q=q_0} \tag{12}$$

is called the charging frequency of the dust particle. This is a measure of how rapidly the charge on the dust particle can change at given conditions.

Equations (10) and (11) give a charge density fluctuation due to the changing charge on the particles which does not exist except in a dusty plasma. The Laplace-Fourier transform of them, which will be used in the following, is

$$\Delta\rho_d(\mathbf{k},s)=\int\delta f_d^\rho(\mathbf{k},\mathbf{v}_d,s)d\mathbf{v}_d \tag{13}$$

and

$$\delta f_d^\rho(\mathbf{k},\mathbf{v}_d,s)=\frac{\delta f_d^\rho(\mathbf{k},\mathbf{v}_d,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_d+\Omega_c} - \frac{\Phi_d(\mathbf{v}_d)}{s+i\mathbf{k}\cdot\mathbf{v}_d+\Omega_c} \times\sum_\alpha\int\sigma_\alpha(q_0,v_\alpha)v_\alpha\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,s,)d\mathbf{v}_\alpha\ . \tag{14}$$

III. DENSITY FLUCTUATION AND CORRELATION FUNCTION

First, some general properties of correlation function are introduced. The relation between a correlation function of the Fourier transform and the spectral function is

$$\lim_{\eta\to 0}2\eta\langle A(\mathbf{k}_1,-i\omega+\eta)B(\mathbf{k}_2,i\omega+\eta)\rangle =(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)\langle AB^*\rangle_{\omega,\mathbf{k}_1} \equiv(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)\langle A^*B\rangle_{\omega,\mathbf{k}_2}\ . \tag{15}$$

The fluctuations we consider are

$$\delta f_\alpha(X,t)=N_\alpha(X,t)-F_\alpha(X,t)\ , \tag{16}$$

X is the sum of all variables of the phase space in which the distribution functions are described. The correlation function of the fluctuations is

$$\langle\delta f_\alpha(X_1,t)\delta f_\beta(X_2,t)\rangle=\delta(X_1-X_2)\delta_{\alpha\beta}F_\alpha(X_1,t) + G_{\alpha\beta}(X_1,X_2,t)\ , \tag{17}$$

in which F_α is the single particle distribution, and $G_{\alpha\beta}$ is the pair correlation function of the α and β species.

The dynamic equation of the plasma particles is

$$\frac{\partial}{\partial t}N_\alpha+\mathbf{v}_\alpha\cdot\frac{\partial}{\partial\mathbf{r}}N_\alpha+\frac{q}{m_\alpha}\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{v}_\alpha}N_\alpha=\nu_\alpha N_\alpha+\mathcal{J}_\alpha\ , \tag{18}$$

where

$$\nu_\alpha=-\int\sigma_\alpha(q,v_\alpha)v_\alpha N_d(\mathbf{v}_d,q)d\mathbf{v}_d dq\ .$$

The first term on the right hand side of Eq. (18) represents the loss rate of the plasma particles in the dust charging collision, and the second one $\mathcal{J}_\alpha$ is a source term.

In many cases in space and in laboratory experiments the dust cloud can be considered to be embedded in an infinite plasma. Examples of this are planetary rings in space, dust in a plasma sheath above a solid surface in an etching process, and dust in other microelectronics plasma processes [11]. In these cases the external source can balance the loss of plasma in the charging collisions as assumed in the Introduction. The dynamic equation for the plasma is

$$\frac{\partial}{\partial t}N_\alpha+\mathbf{v}_\alpha\cdot\frac{\partial}{\partial \mathbf{r}}N_\alpha+\frac{e_\alpha}{m_\alpha}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}_\alpha}N_\alpha=0 . \tag{19}$$

As in Sec. II, here $\langle N_\alpha\rangle=F_\alpha$, $N_\alpha=F_\alpha+\delta f_\alpha$. With the condition $\delta f_\alpha \ll F_\alpha$ the linearization of Eq. (19) gives

$$\frac{\partial}{\partial t}\delta f_\alpha+\mathbf{v}_\alpha\cdot\frac{\partial}{\partial \mathbf{r}}\delta f_\alpha+\frac{e_\alpha}{m_\alpha}\delta\mathbf{E}\cdot\frac{\partial F_\alpha}{\partial \mathbf{v}_\alpha}=0 . \tag{20}$$

From the Laplace-Fourier transform of this equation we get

$$\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,s)=\frac{\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}-\frac{e_\alpha}{m_\alpha}\frac{\delta\mathbf{E}(\mathbf{k},s)}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}\cdot\frac{\partial F_\alpha}{\partial \mathbf{v}_\alpha} . \tag{21}$$

The electric field fluctuation $\delta\mathbf{E}(\mathbf{k},s)$ should be derived from Maxwell equations, for which the Laplace-Fourier's transforms are

$$i\mathbf{k}\cdot\delta\mathbf{E}(\mathbf{k},s)=4\pi\left[\sum_\alpha e_\alpha\int\delta f_\alpha(\mathbf{k},s,\mathbf{v}_\alpha)d\mathbf{v}_\alpha+\int q\,\delta f_d(\mathbf{k},s,q,\mathbf{v}_d)dq\,d\mathbf{v}_d\right] ,$$

$$i\mathbf{k}\cdot\delta\mathbf{B}(\mathbf{k},s)=0 ,$$

$$i\mathbf{k}\times\delta\mathbf{E}(\mathbf{k},s)=-\frac{1}{c}[s\delta\mathbf{B}(\mathbf{k},s)-\mathbf{b}(\mathbf{k})] ,$$

and

$$i\mathbf{k}\times\delta\mathbf{B}(\mathbf{k},s)=\frac{1}{c}[s\delta\mathbf{E}(\mathbf{k},s)-\mathbf{e}(\mathbf{k})]+\frac{4\pi}{c}\left[\sum_\alpha e_\alpha\int\mathbf{v}_\alpha\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,s)d\mathbf{v}_\alpha+\int q\mathbf{v}_d\delta f_d(\mathbf{k},\mathbf{v}_d,q,s)dq\,d\mathbf{v}_d\right] . \tag{22}$$

Here $\mathbf{e}(\mathbf{k})$ and $\mathbf{b}(\mathbf{k})$ are the Fourier's components of the initial electric and magnetic field, respectively. From Eq. (22) we get

$$k^2\delta\mathbf{E}-\mathbf{k}(\mathbf{k}\cdot\delta\mathbf{E})=-\frac{s}{c^2}[s\delta\mathbf{E}-\mathbf{e}]-\frac{i\mathbf{k}\times\mathbf{b}}{c}-\frac{4\pi s}{c^2}\delta\mathbf{j}(\mathbf{k},s) , \tag{23}$$

$$\delta\mathbf{j}(\mathbf{k},s)=\sum_\alpha e_\alpha\int\mathbf{v}_\alpha\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,s)d\mathbf{v}_\alpha+\int q\mathbf{v}_d\delta f_d(\mathbf{k},\mathbf{v}_d,q,s)dq\,d\mathbf{v}_d . \tag{24}$$

First, we consider the dust particles as a component of the plasma, and that they have a mass M_d and a constant charge q_0. Using Eqs. (6) and (20) to calculate $\delta f_\alpha(\mathbf{k},s)$ and $\delta f_d(\mathbf{k},s)$ where we integrate along the undisturbed orbits of the particles as in a normal plasma, we get from Eqs. (23) and (24) that

$$\overleftrightarrow{\Lambda}\cdot\delta\mathbf{E}_0=\mathbf{J}_0(\mathbf{k},s) . \tag{25}$$

Here the Maxwell's tensor $\overleftrightarrow{\Lambda}=\epsilon_L\overleftrightarrow{\Lambda}_L+\Lambda_T\overleftrightarrow{\Lambda}_T$, $\overleftrightarrow{\Lambda}_L=\mathbf{kk}/k^2$, $\overleftrightarrow{\Lambda}_T=\overleftrightarrow{I}-\overleftrightarrow{\Lambda}_L$, $\Lambda_T=\epsilon_T+c^2k^2/s^2$, $\overleftrightarrow{I}$ is a unit tensor, ϵ_L and ϵ_T are the longitudinal and transverse dielectric constants of the dusty plasma, respectively, and

$$\mathbf{J}_0(\mathbf{k},s)=-\frac{4\pi}{s}\left[\sum_\alpha e_\alpha\int\frac{\mathbf{v}_\alpha\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}d\mathbf{v}_\alpha+q_0\int\frac{\mathbf{v}_d\delta f_d(\mathbf{k},\mathbf{v}_d,q_0,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_d}d\mathbf{v}_d\right]+\frac{ic\mathbf{k}\times\mathbf{b}(\mathbf{k})}{s^2}+\frac{\mathbf{e}(\mathbf{k})}{s} . \tag{26}$$

Now let us consider the effect due to the charge changing of dust particles. An additional fluctuation of the electromagnetic field should be produced by the current and charge density fluctuation connected with the changing charge. They are

$$\Delta\mathbf{j}_d(\mathbf{k},s)=\int(q-q_0)\mathbf{v}_d\delta f_d(\mathbf{k},\mathbf{v}_d,q,s)d\mathbf{v}_d\,dq , \tag{27}$$

$$\Delta\rho_d(\mathbf{k},s)=\int(q-q_0)\delta f_d(\mathbf{k},\mathbf{v}_d,q,s)d\mathbf{v}_d\,dq\equiv\int\delta f_d^{\rho}(\mathbf{k},\mathbf{v}_d,s)d\mathbf{v}_d . \tag{28}$$

The effects of the current fluctuations described by Eq. (27) will be neglected because it has no effect on the longitudinal field which is connected with the density fluctuation. While the charge density fluctuation of Eq. (28) has been calculated in Sec. II and given by Eqs. (13) and (14).

According to the Poisson equation an additional longitudinal electric field fluctuation produced by the charge density fluctuation of Eq. (28) is

$$\Delta\delta\mathbf{E}(\mathbf{k},s)=\frac{-i4\pi\mathbf{k}}{k^2}\Delta\rho_d(\mathbf{k},s) .$$

We rewrite Eq. (25) for the electric field fluctuation in the dusty plasma as

$$\overleftrightarrow{\Lambda}\cdot\left[\delta\mathbf{E}+\frac{i4\pi\mathbf{k}}{k^2}\Delta\rho(\mathbf{k},s)\right]=\mathbf{J}_0(\mathbf{k},s) . \tag{29}$$

The longitudinal electric field fluctuation derived from Eqs. (26) and (29) is

$$\delta \mathbf{E}_L(\mathbf{k},s)=\frac{1}{\epsilon_L}\left\{\frac{-i4\pi\mathbf{k}}{k^2}\left[\sum_\alpha e_\alpha \int \frac{\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}d\mathbf{v}_\alpha+\int q_0\frac{\delta f_d(\mathbf{k},\mathbf{v}_d,q_0,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_d}d\mathbf{v}_d\right]\right\}-\frac{i4\pi\mathbf{k}}{k^2}\Delta\rho_d(\mathbf{k},s) \ . \quad (30)$$

Substituting Eq. (30) into (21), we get

$$\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,s)\equiv\delta f_\alpha^{\mathrm{I}}(\mathbf{k},\mathbf{v}_\alpha,s)+\delta f_\alpha^{\mathrm{II}}(\mathbf{k},\mathbf{v}_\alpha,s)$$

$$=\left\{\frac{\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}+\frac{1}{\epsilon_L}\frac{i4\pi e_\alpha}{k^2 m_\alpha}\frac{\mathbf{k}\cdot\partial F_\alpha/\partial\mathbf{v}_\alpha}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}\right.$$

$$\left.\times\left[\sum_{\beta=e,i} e_\beta\int\frac{\delta f_\beta(\mathbf{k},\mathbf{v}_\beta,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_\beta}d\mathbf{v}_\beta+\int q_0\frac{\delta f_d(\mathbf{k},\mathbf{v}_d,q_0,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_d}d\mathbf{v}_d\right]\right\}$$

$$+\left\{\frac{i4\pi e_\alpha}{k^2 m_\alpha}\frac{\mathbf{k}\cdot\partial F_\alpha/\partial\mathbf{v}_\alpha}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}\Delta\rho_d(\mathbf{k},s)\right\}, \quad (31)$$

where $\delta f_\alpha^{\mathrm{I}}$ and $\delta f_\alpha^{\mathrm{II}}$ refer to the first and second brackets of Eq. (31), respectively. Also, the density fluctuation can be written as

$$\delta n_\alpha(\mathbf{k},s)=\int\delta f_\alpha(\mathbf{k},s,\mathbf{v}_\alpha)d\mathbf{v}_\alpha\equiv\delta n_\alpha^{\mathrm{I}}(\mathbf{k},s)+\delta n_\alpha^{\mathrm{II}}(\mathbf{k},s) \ . \quad (32)$$

Substituting Eq. (31) into (32) and using Eqs. (13) and (14), we get

$$\delta n_\alpha^{\mathrm{I}}(\mathbf{k},s)=\int\frac{\delta f_\alpha(\mathbf{k},\mathbf{v}_\alpha,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}d\mathbf{v}_\alpha-\frac{4\pi\chi_{L\alpha}(\mathbf{k},s)}{\epsilon_L}$$

$$\times\left[\sum_{\beta=e,i}\frac{e_\beta}{e_\alpha}\int\frac{\delta f_\beta(\mathbf{k},\mathbf{v}_\beta,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_\beta}d\mathbf{v}_\beta+\int\frac{q_0}{e_\alpha}\frac{\delta f_d(\mathbf{k},\mathbf{v}_d,q_0,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_d}d\mathbf{v}_d\right], \quad (33)$$

$$\chi_{L\alpha}(\mathbf{k},s)=-\frac{ie_\alpha^2}{k^2 m_\alpha}\int\frac{\mathbf{k}\cdot\partial F_\alpha/\partial\mathbf{v}_\alpha}{s+i\mathbf{k}\cdot\mathbf{v}_\alpha}d\mathbf{v}_\alpha \ , \quad (34)$$

and

$$\delta n_\alpha^{\mathrm{II}}(\mathbf{k},s)=-\frac{4\pi\chi_{L\alpha}(\mathbf{k},s)}{e_\alpha}\left[\int\frac{\delta f_d^\rho(\mathbf{k},\mathbf{v}_d,t=0)}{s+i\mathbf{k}\cdot\mathbf{v}_d+\Omega_c}d\mathbf{v}_d\right.$$

$$\left.-\left[\int\frac{\Phi_d(\mathbf{v}_d)}{s+i\mathbf{k}\cdot\mathbf{v}_d+\Omega_c}d\mathbf{v}_d\right]\left[\sum_{\beta=e,i}e_\beta\int\sigma_\beta(q_0,\mathbf{v}_\beta)v_\beta\delta f_\beta(\mathbf{k},s,\mathbf{v}_\beta)d\mathbf{v}_\beta\right]\right]. \quad (35)$$

In order to derive the dynamic spectral density fluctuation we first find some related correction functions which are calculated based on Eqs. (15) and (17). They are

$$S_\alpha^{\mathrm{I}}(\omega,\mathbf{k})=\frac{1}{n_\alpha}\langle\delta n_\alpha^{\mathrm{I}}\delta n_\alpha^{\mathrm{I}*}\rangle_{\omega,\mathbf{k}}$$

$$=\frac{2\pi}{n_\alpha}\left\{\left|1-\frac{4\pi\chi_{L\alpha}(\omega,\mathbf{k})}{\epsilon_L(\omega,\mathbf{k})}\right|^2\int\delta(\omega-\mathbf{k}\cdot\mathbf{v}_\alpha)F_\alpha(\mathbf{v}_\alpha)d\mathbf{v}_\alpha+\sum_{\beta\neq\alpha}\frac{e_\beta^2}{e_\alpha^2}\left|\frac{4\pi\chi_{L\alpha}(\omega,\mathbf{k})}{\epsilon_L(\omega,\mathbf{k})}\right|^2\int\delta(\omega-\mathbf{k}\cdot\mathbf{v}_\beta)F_\beta(\mathbf{v}_\beta)d\mathbf{v}_\beta\right.$$

$$\left.+\frac{q_0^2}{e_\alpha^2}\left|\frac{4\pi\chi_{L\alpha}(\omega,\mathbf{k})}{\epsilon_L(\omega,\mathbf{k})}\right|^2\int\delta(\omega-\mathbf{k}\cdot\mathbf{v}_d)\Phi_d(\mathbf{v}_d)d\mathbf{v}_d\right\}$$

$$\equiv S_{\alpha1}^{\mathrm{I}}+S_{\alpha2}^{\mathrm{I}}+S_{\alpha3}^{\mathrm{I}} \ . \quad (36)$$

Here $S_{\alpha I}^{\mathrm{I}}$, $S_{\alpha2}^{\mathrm{I}}$, and $S_{\alpha3}^{\mathrm{I}}$ refer to the first, second, and third terms in the equation, respectively.

$$
\begin{aligned}
S_\alpha^{\rm II}(\omega,\mathbf{k}) &= \frac{1}{n_\alpha}\langle \delta n_\alpha^{\rm I}\delta n_\alpha^{\rm II*}\rangle_{\omega,\mathbf{k}} + \frac{1}{n_\alpha}\langle \delta n_\alpha^{\rm I*}\delta n_\alpha^{\rm II}\rangle_{\omega,\mathbf{k}} \\
&\simeq \frac{n_d}{n_\alpha}\frac{16\pi^2\Omega_c}{\omega^2+\Omega_c^2}\mathrm{Re}[\chi_{L\alpha}(\omega,\mathbf{k})] \\
&\quad\times\left\{\left[1-\frac{4\pi\,\mathrm{Re}[\chi_{L\alpha}(\omega,\mathbf{k})]}{\mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]}\right]\left[\sum_{\beta=e,i}\int \sigma_\beta(q_0,v_\beta)v_\beta\delta(\omega-\mathbf{k}\cdot\mathbf{v}_\beta)F_\beta d\mathbf{v}_\beta\right]\right. \\
&\qquad+\left[\sum_{\beta=e,i}\frac{4\pi e_\alpha e_\beta}{k^2 m_\beta \mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]}\int \sigma_\beta(q_0,v_\beta)v_\beta\frac{\mathbf{k}\cdot\partial F_\beta/\partial\mathbf{v}_\beta}{\omega-\mathbf{k}\cdot\mathbf{v}_\beta}d\mathbf{v}_\beta\right] \\
&\qquad\times\left[\left[\frac{4\pi\,\mathrm{Re}[\chi_{L\alpha}(\omega,\mathbf{k})]}{\mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]}-1\right]\int\delta(\omega-\mathbf{k}\cdot\mathbf{v}_\alpha)F_\alpha(\mathbf{v}_\alpha)d\mathbf{v}_\alpha\right. \\
&\qquad\quad+\frac{4\pi\,\mathrm{Re}[\chi_{L\alpha}(\omega,\mathbf{k})]}{\mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]}\left[\sum_{\beta\neq\alpha}\frac{e_\beta^2}{e_\alpha^2}\int\delta(\omega-\mathbf{k}\cdot\mathbf{v}_\beta)F_\beta(\mathbf{v}_\beta)d\mathbf{v}_\beta\right] \\
&\qquad\quad\left.\left.+\frac{4\pi\,\mathrm{Re}[\chi_{L\alpha}(\omega,\mathbf{k})]}{\mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]}\frac{q_0^2}{e_\alpha^2}\int\delta(\omega-\mathbf{k}\cdot\mathbf{v}_d)\Phi_d(\mathbf{v}_d)d\mathbf{v}_d\right]\right\} \\
&\equiv S_{\alpha1}^{\rm II}+S_{\alpha2}^{\rm II}\ .
\end{aligned}
\tag{37}
$$

Here $S_{\alpha1}^{\rm II}$ and $S_{\alpha2}^{\rm II}$ refer to the first and second parts, respectively.

In the above derivation the following relations have been used:

$$
\begin{aligned}
&\epsilon_L^*(-\omega,-\mathbf{k})=\epsilon_L(\omega,\mathbf{k})\ , \\
&\chi_{L\alpha}^*(-\omega,-\mathbf{k})=\chi_{L\alpha}(\omega,\mathbf{k})\ ,
\end{aligned}
\tag{38}
$$

$$
\lim_{\eta\to 0}\int\frac{2\eta\psi(x)}{(x-i\eta)(x+i\eta)}dx=i2\pi\int\delta(x)\psi(x)dx \tag{39}
$$

and the inequality

$$
\mathrm{Re}[\chi_{L\alpha}(\omega,\mathbf{k})]\gg\mathrm{Im}[\chi_{L\alpha}(\omega,\mathbf{k})]\ . \tag{40}
$$

IV. DYNAMIC SPECTRAL DENSITY FUNCTION OF A DUSTY PLASMA

A ratio of Eqs. (36) and (37) leads to

$$
\Gamma=\left|\frac{S_\alpha^{\rm II}(\omega,\mathbf{k})}{S_\alpha^{\rm I}(\omega,\mathbf{k})}\right|\simeq P\left[\frac{\Omega_c}{\omega}\right]^2 . \tag{41}
$$

Here P is a dusty plasma parameter, which describes the importance of the dust charge in the plasma,

$$
P=\frac{n_d a k_B T_e}{n_0 e^2}\ , \tag{42}
$$

n_0 is the density of the background plasma in which the dust is embedded and k_B is Boltzmann's constant [12]. A dusty plasma is tenuous in dust when $P\ll 1$, and dense when $P\gg 1$. The latter is a case of a strong Coulomb interaction of dust which we are not concerned with here. For a Maxwellian plasma the charging frequency, which is found from Eqs. (4) and (12), is

$$
\Omega_c=\frac{a}{\sqrt{2\pi}}\left[\frac{\omega_{pe}^2}{v_{Te}}\exp\left[-\frac{\mathcal{U}_e^2}{v_{Te}^2}\right]+\frac{\omega_{pi}^2}{v_{Ti}}\right]\ , \tag{43}
$$

where we have used an abbreviation

$$
\mathcal{U}_\alpha^2=\frac{2|q_0|e}{m_\alpha a}\ .
$$

In many situations in space and in experiments $\Gamma\ll 1$ due to $P\ll 1$ or $\Omega_c/\omega\ll 1$.

Neglecting the terms of higher order of Γ, the dynamic spectral density fluctuation should be

$$
\begin{aligned}
S_\alpha(\omega,\mathbf{k})&=S_\alpha^{\rm I}(\omega,\mathbf{k})+S_\alpha^{\rm II}(\omega,\mathbf{k}) \\
&\equiv S_{\alpha1}^{\rm I}+S_{\alpha2}^{\rm I}+S_{\alpha3}^{\rm I}+S_{\alpha1}^{\rm II}+S_{\alpha2}^{\rm II}\ .
\end{aligned}
\tag{44}
$$

The expressions of all terms in Eq. (44) have been given by Eqs. (36) and (37) in Sec. III. In the following we make a detailed study of the electron dynamic spectral density function in a dusty plasma. Setting $\alpha=e$ in Eq. (44) and related Eqs. (36) and (37), obviously we get a result which differs from that of a normal plasma. This means that the collective interaction in a dusty plasma is greatly changed by the existence of the dust particles. Some points are summarized below.

(i) The term $S_e^{\rm I}(\omega,\mathbf{k})$ is the effect of the Coulomb interaction of a dusty plasma in which the dust particles have a constant charge q_0. Here $S_{e1}^{\rm I}$ is the effect of bare-electron Coulomb interaction, $S_{e2}^{\rm I}$ represents an effect of dressed ions, while the term $S_{e3}^{\rm I}$ comes from the dressed dust particles.

(ii) Equation (35) shows that $S_{e3}^{\rm I}$ is proportional to the density ratio n_d/n_e and the ratio of charge square of the dust charge number $Z_d^2=(q_0/e)^2$. In most cases, $S_3^{\rm I}$ can make an important contribution to $S_e^{\rm I}(\omega,\mathbf{k})$ since the charge on the dusty particle is high. For a tenuous $(P\ll 1)$ hydrogen plasma

$$Z_d = 2.51 \frac{k_B T a}{e^2} = 1.75 \times 10^3 [T \text{ (eV) } \alpha \text{ } (\mu\text{m})] \, .$$

For such charges the Coulomb screening effect of the dust particles increases the electron density correlation greatly.

(iii) The term $S_e^{\rm II}(\omega,\mathbf{k})$ describes the effect due to the charge changing of dust particles. It is related to the dust particle density n_d and the charging frequency of the dusty particle Ω_c. As shown in Eq. (41), $S_e^{\rm II}(\omega,\mathbf{k})$ gives a correction of order $P\Omega_c^2/\omega^2$ smaller than $S_e^{\rm I}(\omega,\mathbf{k})$.

(iv) We evaluate $S_e^{\rm II}(\omega,\mathbf{k})$ in the long wavelength limit $(k \to 0)$. One of the coefficients in the expression for $S_e^{\rm II}(\omega,\mathbf{k})$ [see Eq. (37)] can be calculated approximately,

$$\begin{aligned}
&\sum_{\beta=e,i} \frac{-4\pi e e_\beta}{k^2 m_\beta \mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]} \int \sigma_\beta(q_0, v_\beta) v_\beta \frac{\mathbf{k}\cdot\partial F_\beta/\partial \mathbf{v}_\beta}{\omega - \mathbf{k}\cdot\mathbf{v}_\beta} d\mathbf{v}_\beta \\
&\quad \simeq \sum_{\beta=e,i} \frac{4\pi e e_\beta}{k^2 m_\beta v_{T\beta}^2 \omega^2 \mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]} \int \sigma_\beta(q_0, v_\beta) v_\beta (\mathbf{k}\cdot\mathbf{v}_\beta)^2 F_\beta(\mathbf{v}_\beta) d\mathbf{v}_\beta \\
&\quad = \frac{-2\sqrt{\pi} a^2}{3\,\mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]} \left[\frac{\omega_{pe}^2}{\omega^2} v_{Te} \left[1 - \frac{\mathcal{U}_e^2}{2v_{Te}^2} \right] \exp\left[-\frac{\mathcal{U}_e^2}{v_{Te}^2} \right] - \frac{\omega_{pi}^2}{\omega^2} v_{Ti} \left[1 + \frac{\mathcal{U}_i^2}{2v_{Ti}^2} \right] \right] \\
&\quad \simeq \frac{2\sqrt{\pi} a^2 v_{Ti}}{3\,\mathrm{Re}[\epsilon_L(\omega,\mathbf{k})]} \frac{\omega_{pi}^2}{\omega^2} \frac{v_{Te}^2}{v_{Ti}^2} \left[1 + \frac{\mathcal{U}_i^2}{2v_{Ti}^2} \right] \left[\frac{\mathcal{U}_e^2}{2v_{Te}^2} - 1 \right] .
\end{aligned} \tag{45}$$

We have used the inequality $\omega/k \gg v_{T\alpha}$, the expression for the collision cross section Eq. (5), and an equation which follows from Eq. (7); it is

$$\frac{\omega_{pe}^2}{v_{Te}} \exp\left[-\frac{\mathcal{U}_e^2}{v_{Te}^2} \right] = \frac{\omega_{pi}^2}{v_{Ti}} \left[1 + \frac{\mathcal{U}_i^2}{2v_{Ti}^2} \right] ,$$

where the plasma particles have been assumed to have a Maxwellian distribution with temperature T_α.

Substituting Eq. (45) into Eq. (37), we find in the case of $\omega \gg \omega_{pe}$

$$S_e^{\rm II}(\omega,\mathbf{k}) = P\zeta \frac{\Omega_c^2}{\omega^2 + \Omega_c^2} \frac{\omega_{pi}^2 \omega_{pe}^2}{\omega^4} \times \left[\frac{1}{k v_{Te}} \exp\left[-\frac{\omega^2}{v_{Te}^2 k^2} \right] \right] , \tag{46}$$

where for $\omega \ll \omega_{pe}$ we find

$$S_e^{\rm II}(\omega,\mathbf{k}) = -P\zeta \frac{\Omega_c^2}{\omega^2 + \Omega_c^2} \frac{\omega_{pi}^2 \omega_{pe}^2}{\omega^4} \times \left[\frac{n_i}{n_e} \frac{1}{k v_{Ti}} \exp\left[-\frac{\omega^2}{v_{Ti}^2 k^2} \right] + \frac{n_d}{n_e} \left[\frac{q_0}{e} \right]^2 \frac{1}{k v_{Td}} \exp\left[-\frac{\omega^2}{v_{Td}^2 k^2} \right] \right] . \tag{47}$$

Here the abbreviation

$$\zeta = \frac{64\sqrt{2}\pi^{5/2}}{9} \frac{n_0}{n_{i0}} \frac{m_i}{m_e} \left[\frac{\mathcal{U}_e^2}{2v_{Te}^2} - 1 \right] \times \left[1 + \frac{\mathcal{U}_e^2}{2v_{Te}^2} \frac{T_e}{T_i} \right] \left[1 + \frac{T_i}{T_e} + \frac{\mathcal{U}_e^2}{2v_{Te}^2} \right]^{-1} .$$

For a tenuous hydrogen plasma

$$\zeta = 3.24 \times 10^5 \frac{n_0}{n_{i0}} \left[1 + 1.255 \frac{T_e}{T_i} \right] \times \left[\frac{T_i}{T_e} + 2.255 \right]^{-1} .$$

It can be seen from Eqs. (46) and (47) that there is a positive correction of $S_e^{\rm II}(\omega,\mathbf{k})$ for the high frequency fluctuation and a negative one for the low frequency fluctuation due to the charge changing of dust particles.

(v) The dust particles have affects on dielectric properties and also changes the collective modes, especially for the modes of low frequency. In some cases the inequality (40) does not hold, for example, when the charging frequency of the dust particle of Eq. (12) is compared with the wave frequency. So the fluctuation of low frequency in a dusty plasma, which are associated with the collective modes, will be different. A detailed study of this will be presented in another paper. The result is that a strong fluctuation exists if the charging frequency of the dusty particle is much smaller than the frequency of the dust acoustic waves; otherwise the fluctuation would be weak.

(vi) The power spectrum of scattered waves in a plasma is directly proportional to the dynamic spectral density function $S_e(\omega,\mathbf{k})$ of the plasma. The results here mean that dust particles in a plasma can change the wave scattering in a plasma substantially. As a result, scattering experiments, in addition to measuring the density fluctuation spectrum of the plasma, also obtain valuable information about the dust.

V. CONCLUSION AND DISCUSSION

It is well known that there is a rich class of phenomena in plasmas which are related to their statistical properties. In the present paper the dynamic spectral density function of a dusty plasma in a weak turbulence and weakly coupled state has been studied. The result shows that the dusty grains change the electron density correlation of the plasma in two ways: (1) a great increase of the correlation due to the Debye screening of the dust particles, and (2) a positive correction for the high frequency fluctuation and a negative correction for low frequency fluctuation due to the charge changing of dust particles.

In the above study the assumption of weak coupling means that a condition of the Coulomb coupling parameter

$$\Gamma_c = \frac{(Z_d e)^2}{d k_B T} \ll 1 ,$$

should be satisfied, where

$$d = \left[\frac{4\pi n_d}{3} \right]^{-1/3}$$

is the radius of a sphere with the characteristic volume $1/n_d$. This condition for a tenuous hydrogen plasma is

$$\Gamma_c = 7.1 \times [T\ (\mathrm{eV})][a^2\ (\mu\mathrm{m})^2] \left[\frac{n_d}{[10^3\ (\mathrm{cm}^{-3})]} \right]^{1/3} \ll 1 .$$

It is valid for the cases of small grains with low temperature. Such cases could, for example, be the dust cloud in space plasma. In other cases especially in the laboratory the dusty plasma is in a strongly coupled state. The weakly collision approach which has been used in this paper could be invalid for them. But the results of this paper show that the Debye screening of the dust particles and the charge changing of the dust particles should make significant effects on the collective properties of the plasma.

ACKNOWLEDGMENTS

This work was supported by NSF Grant of China. The author is indebted to Professor O. Havnes, University of Tromsø, Norway, for his careful reading of the paper.

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